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ABSTRACT

The role of writing in the mathematics classroom is considered. Writing as a cognitive process and its relationship to problem solving is first examined. Then several long- and short-term techniques for promoting cognitive learning using writing in the mathematics classroom are considered. The Writing Across the Curriculum (WAC) program is discussed; some of its techniques can be adapted to encourage problem solving and cognitive learning. Suggested for the long term are: (1) a list of writing projects requiring several days or weeks to complete; and (2) a review method involving whole-class, individual, and group work. Short-term techniques usually require only a few minutes each day, but may take a whole class period. Discussed are explaining how, comparing two concepts, explaining why, doing word problems, outlining the chapter, developing test questions, doing proofs with prose, and keeping a notebook. Finally, how to get started is discussed, with note of the need for persistence. (MNS)

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WRITING IS PROBLEM SOLVING

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Writing IS Problem Solving

Russel W. Kenyon

The development of cognitive theory in the 1960s and 1970s has led to the structuring of the general area in mathematics called problem solving. Several educational institutions have followed the lead of the National Council of Teachers of Mathematics' (NCTM, 1980a) policy statement that "problem solving be the focus of school mathematics in the 1980s" (p.1). Only recently, however, have mathematics educators, educational psychologists, and cognitive psychologists begun to work together to develop meaningful programs of study that deal with the cognitive learning skills. These skills can be acquired in mathematics learning through the use of several methods of problem solving, including group problem solving, writing, and various uses of the computer and manipulatives. Of these several methods, writing has been used the least in mathematics classrooms.

Why would anyone want to use writing in a mathematics classroom? This paper will attempt to answer that question by first defining cognitive learning and then examining its relationship to problem solving. Writing will be looked at as an important problem solving method and it will be shown that the use of writing in mathematics classrooms will result in cognitive learning.

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We will then consider several long-term and short-term techniques for promoting cognitive learning using writing in the mathematics classroom.

Writing for Problem Solving: A Cognitive Process

Mathematics instructors have traditionally seen their task as one of presenting material to their students in a clear and precise manner. The model student, having heard the truth from the "expert" standing before him or her, copies the rules and definitions, and memorizes them. This has been referred to as the "copy theory" of learning. In general, students feel very comfortable with this strategy. However, students' adherence to this strategy "may be the single most important deterrent to effective education" (Lochhead, 1985).

Significant changes are being made in mathematics classrooms around the nation. New methods are proving effective in bringing passive students into the learning process in an active way, thereby increasing the efficiency of their learning. Students who simply copy and memorize are being replaced by those who are actively involved in the learning process and are "constructing" their own knowledge structure as they proceed. This active involvement is readily observable. In a recent study (Konold, 1986), time-on-task measurements were taken on students while they were learning by using the lecture method, and while they

were learning by using the "pair" or "small group" problem solving approach. (In pair problem solving, one student is the problem solver, and the other is the listener-observer who asks for explanations of the problem solving process while the problem solver is solving a word problem.) The pair problem solving method yielded a much more sustained student effort (r=-.142) than did the lecture method (r=-.424).

More active students also have a different attitude toward their work. They are more willing to look for new approaches to problems, to develop new algorithms, and in general, to spend more time-on-task. Passive students, on the other hand, are ready to give up if the solution is not immediately at hand. They believe that you either know the solution or you don't. Bloom and Broder (1950) report that the major difference between successful and unsuccessful problem solvers is the degree to which they are active or passive in their approach to a problem.

Verbalization is a technique that requires the learner to report on the thinking process during problem solving. It is based on the assumption that the problem solving process will be understood by the learner only when he or she can explain it to others. In the case of small or large group problem solving, most of the verbalization is spoken. Thus, the processing is available to both the problem solver and to others to observe and evaluate. Higher level skills

are called upon to coherently explain each step in the process so that all will understand the solution.

Writing down the thoughts and procedures involved for each of the steps of a problem solution adds yet another dimension to the processing. The problem solver can now clearly "see" the steps of the solution described in written words and has immediate feedback for review and reflection. Writing encourages the evaluation and modification of each of the steps. Learning is thus enhanced as mathematical facts and theorems are recalled and organized on paper. This allows for further analysis, evaluation, and possible modification to take place. This learning process needs to be further investigated to understand how it is affected by verbalization, writing, and other problem solving methods.

Cognitive Learning

Cognitive scientists have recently studied learning by constructing a complex model of the human memory system (Anderson, 1983). Some of the modern technologies, such as computer science, information processing, and linguistics, have been applied to this work. The model has been tested and modified, and is a relatively good representation of the human memory system. Based on this model, which is called Adaptive Control of Thought (ACT), learning can now be defined in terms of two different types of information acquisition. The first type involves the addition of

propositions to declarative memory, and the second, the construction and addition to production memory of procedures which originate from other procedures and from propositions. The following definition of learning is based on this ACT model: Learning is the acquisition of knowledge and skills, where knowledge refers to the propositions, spatial images, and temporal strings stored in declarative memory, and skills refer to the procedures stored in production memory.

Process of Problem Solving

Cognitive learning and problem solving are closely related. Now that we have a definition for learning, let us carefully define problem solving. But first we need to understand what a "problem" is. Bell (1978) gives four characteristics of a problem:

1. A person must be aware of a situation.
2. The person must recognize that the situation requires action.
3. The person must either want or need to act upon the situation and must actually take some action.
4. The resolution of the situation must not be immediately obvious to the person acting on it.

It is important to note that in this definition, the person does not have an algorithm available to resolve the

situation. Expressing it in cognitive theory terms, there is no propositional knowledge in declarative memory, or procedural knowledge in production memory which would allow the person to move directly from the problem state to the goal state. Problem solving can now be defined as the process by which a person uses previously acquired knowledge and skills to attempt to find a resolution, not immediately apparent, to a situation (problem) which confronts him or her.

Note that problem solving is not simply obtaining the final answer. Rather, it is the entire process, starting with the original confrontation of the situation, through the various steps in the general problem solving procedure, to a conclusion. Problem solving utilizes knowledge and skills in declarative and production memory to compile new knowledge which is then added to memory. Therefore, problem solving always yields cognitive learning.

Problem solving involves application of learned knowledge and skills in order to move through the process of resolving a situation which confronts a person and which the person needs or wants to resolve. The person, when confronted with a situation, first needs to determine what he or she knows about the situation and then determine how to get from this knowledge base to the goal. The next step would be to carry out the plan. The final step would be to evaluate the results and the procedure (Polya, 1957).

Learning takes place during the problem solving process since the new information combines with the existing knowledge and is stored in memory in this new form.

Several rules and procedures, often referred to as heuristics, have been compiled to aid the problem solver in the planning stage (Wickelgren, 1974). General problem solving strategies have also been developed. It is not entirely clear at this time just what problem solving strategies and procedures should be taught or at what level. Certainly, domain-based knowledge can be taught, as can some of the general problem solving strategies. It appears that at least some problem solving skills are acquired by simply doing problem solving (Schoenfeld, 1982,1985).

Writing and Problem Solving

Although problem solving has been the watchword of the 1980s in mathematics education, the method for problem solving in mathematics that has garnered the least attention is writing. The Writing Across the Curriculum (WAC) program which has received support from many school systems in this country and abroad, has not been supported by the mathematics educational community, in my opinion. The WAC literature is clear on the purpose of the program: to teach learning through writing (Tchudi & Yates, 1983). Some of the stages in the WAC program, which will be discussed in the next section, are similar to the steps in problem

solving. It becomes evident when the definition for problem solving is examined and compared to the writing stages in WAC that writing is problem solving.

In the mathematics classroom writing can be used effectively for the acquisition of knowledge and skills through the problem solving process. Asking students to write about a problem requires them to clarify thoughts about how they will approach the problem. This writing makes the concepts about the problem clearer and sharper. The writing process then becomes an integral part of the thought process (McMillen, 1986). In this writing process, students begin to gather, formulate, and organize old and new knowledge, concepts, and strategies, and to synthesize this information as a new structure that becomes a part of their own knowledge network (Nahrgang & Petersen, 1986). Of equal importance to cognitive learning is the acquisition of the metacognitive skills which allow students to perform the self-monitoring functions in problem solving. As students write down, reflect on, and react to their thoughts and ideas, they enhance the executive problem solving abilities, and the problem solving process becomes more effective.

Techniques for Using Writing in Mathematics

To promote this thought processing, activities should be designed to assure that writing becomes a normal part of

the daily routine in the mathematics classroom. The students should begin to gain the necessary writing skills as the writing takes place. They should also begin to expect writing to be a standard part of the mathematics program. The following ideas should help the mathematics teacher make writing an integral part of that program.

Long-Term Writing Techniques

We will briefly consider the stages of writing from the Writing Across the Curriculum (WAC) program. WAC does not directly address the issues of cognitive learning, nor does it appear to contain the applications for mathematics that are specifically designed to encourage the higher-level thinking in mathematics processing. However, some of the WAC techniques may be adapted so as to encourage problem solving, and therefore, cognitive learning.

Writing is normally described as being divided into several stages. Most authors agree on three main stages, while others add additional ones. The first stage is the prewriting or planning stage. The prewriting phase probably requires more cognitive processing than any of the other stages. The student is involved in the initial state of the process of problem solving, and is attempting to understand the conditions of the problem so as to achieve a goal (or the goal state). A memory search takes place, looking for data, strategies and techniques to aid in the solution.

Ideas for possible solutions are generated and organized during this process using notetaking and exploratory freewriting. In the final phase of this stage, all the possible ideas are evaluated and the final candidate(s) are selected.

The next stage is the writing or composition stage. At this point, the ideas are translated into an extended text. The student should not be concerned about the mechanics, i.e., the spelling and punctuation, at this time. If the ideas were all completely formed in the first stage, the writing should be quite automatic. Otherwise, the student must return to the first stage. The last stage, according to some authors is the revising or rewriting stage. This is the point at which the copy should be carefully read and rewritten for clarity and focus. Ideas can be moved around, redundant material removed, and needed information added (Anderson, 1985; Gagne', 1985). The WAC program adds to these stages the copyediting and the presenting and publishing stages (Tchudi & Yates, 1983). Several ideas for long-term writing will be given, as well as an adaptation of a method from the Writing for Learning program that can be applied to the mathematics program.

Long-Term Writing Projects. After the writing program is well established, the teacher may want to assign a writing project which will require several days or weeks to

complete. The project should be carefully designed to assure that it meets the course/unit objectives and is within the students' technical and writing capabilities. Several of these projects require access to libraries, banking information, and other records. The following is a list of ideas for long-term projects in the mathematics classroom:

-Paper on a career in a mathematics-related field.

-Book report on a mathematics or computer related book or paper; e.g., Mindstorms, papers from The Mathematics Experience, Men of Mathematics, or The World of Mathematics.

-Letter to a national testing service, offering suggestions on methods to evaluate and measure intelligence.

-Document outlining a plan to form a financial planning group that would give advice to students on how to earn, save, and manage money.

-Report on statistics as used in the insurance field.

-Paper on a famous mathematician.

-Report on mathematics anxiety.

-Report on how mathematics is used to measure and evaluate intelligence, reading skills, mathematical abilities, and other aptitudes.

-Report on the historical development of some area of mathematics such as algebra.

-Report on a chapter from a mathematics book on a topic not covered in the course this year. Study it, keeping a

journal of your thoughts and feelings as you progress through the unit. Summarize your journal in a report.

-Report on new developments in computer technology.

-Letter to a friend/peer on how he or she feels about mathematics.

-Letter to a friend/peer offering advice on solving a certain problem.

-Discussion of a retired persons budget, considering various expenses and sources of income.

-Paper presenting a solution to a longer mathematics problem. Write out the solution in words (no mathematics), describing each step with its reason.

-Report on the origins of a particular measurement; e.g., length of a day.

-Letter taking the role of a lawyer, dentist, or stockbroker, for example, explaining why one should master the concepts of algebra I.

A Writing for Learning Application. At the end of a major unit, the main concepts usually need to be brought together and organized. A method from the Writing for Learning manual (Wolfe & Reising, 1983) is particularly suitable for this review application. The following is an adaptation of "A Process Approach to Writing for Learning" from that manual. The review of a unit using this technique will require a full 40-minute class period and a follow-up

homework assignment. The method is suitable for a variety of applications.

Assume that a unit on solving linear equations by various methods has just been completed. The class period today will be devoted to a review to prepare for a test which is scheduled for the day after tomorrow. The following is the schedule for today's class period:

1. The reflection stage. 10 minutes. Students work individually, brainstorming on the various methods for solving linear equations. They try to outline the material and to draw any pictures that come to mind. This is the time to jot down all the questions they can think of, and to write out phrases, clauses, and full sentences about the various methods.

2. The clarification stage. 5 minutes. The students come together with the teacher to make sure that all understand the procedure and to ask final questions before going on. The teacher then gives the instructions for the next stage.

3. The first draft stage. 15 minutes. The students again work individually. Everything they know about solving linear equations and about the different methods is written down. There is some time for rewriting, for adding, for deleting, for modifying. Some of the original chaos becomes order.

4. The peer inquiry stage. 10 minutes. Groups of two to five students are formed to share the work from the last stage. Each student's work is read in the group, and the group gives two positive, concrete comments about the writing and then makes two suggestions for revising or editing.

5. The revision stage. The homework assignment for the next day is to rewrite the paper considering peer comments. This stage brings together the methods of solving linear equations in a concrete form.

6. The teacher inquiry stage. During the next class, the teacher reads the papers, making complimentary remarks and offering suggestions. Any misconceptions are corrected.

7. The second revision stage. For a mathematics program, this is an optional step.

Short-Term Writing Techniques.

Ideas for using writing in mathematics have been offered in several recent papers (Birken, 1986; Geeslin, 1977; Johnson, L., 1985; Johnson, M., 1983; King, 1982; Lubatkin, 1986; McMillen, 1986; Nahrgang & Petersen, 1986; Salem, 1982; and Tchudi & Yates, 1984). These ideas fall into several broad categories which make them adaptable to a variety of teaching styles, grade levels, and class sizes. These short-term techniques can be used as writing exercises that require just a few minutes of class time each day, or they may require the whole period.

Explain How. This procedure is similar to the pair problem solving method (Whimbey & Lochhead, 1980, 1984), except that each step is written rather than spoken. Student will need some practice to develop this technique, but it is well worth the effort. See Figure 6.1 (general writing instructions) and Figure 6.2 (writing probes) for ideas to help student become more proficient in this technique. Discuss the

general writing instructions at the beginning of the writing program, and encourage students to use the writing probes whenever using this writing technique. Salem (1982) refers to this technique as "chatter: students talk to themselves on paper as they do a problem" (p. 129).

As an example, say to the students: "Consider that solving an equation is undoing what has been done to a variable. Write what has been done to the variable in the following equation. Then state what must be done to reverse that."

X/3 + 7 = -13	"X is divided by 3, then 7 is added".
Subtract 7	This is the first step to undo that.
X/3 = -20	"X is divided by 3."
Multiply by 3	To undo that.
X = -60	

A similar technique is to ask students to write the steps to a problem solution. Nonmathematical problems work as well as mathematical ones here. For example, ask students to describe how to build a tower using blocks of different sizes and colors. Then have the students exchange their solutions and attempt to rebuild the tower using only the given instructions. Finally, have the students evaluate their solution by discussing the results of the tower construction. The same procedure can be used with a mathematical solution.

Another alternative to this technique is to require that only the solution steps be written out; that is, leave out all reference to the numbers and/or variables. For example, explain how to add 1/3 and 1/4 without mentioning the 1/3 or 1/4. It is usually a good idea to add something to the instructions like "assume you are explaining this

process to a sixth grader." The "explain how" technique is a good one to start the writing program with.

Compare Two Concepts. This technique requires students to first define the meanings of various terms in their own words. They may start by constructing rather poorly written paragraphs. As they are able to express themselves more precisely, the paragraphs will shorten to one or two sentences. After students are able to explain the meaning of individual terms in one or two sentences, they are given pairs of terms representing either similar or contrasting concepts. For example, explain how the following concepts are related: equation-graph, line-plane, number line-coordinates, point-line, linear equation-quadratic equation, fraction-decimal number, circle-ellipse, function-relation, and geometry-algebra. When starting to use this technique, again, students may need to write a paragraph or more. Discourage the use of memorized definitions and concepts. As the technique develops, students should be able to formulate ideas more succinctly, and the paragraph will be reduced to one or two sentences (Geeslin, 1977).

Explain Why. This is a more advanced technique and requires more preparation on the teacher's part. The student should have done some form of writing, for example, the "explain how" method, before using this technique. This technique uses the open-ended question format, so that a final, definitive solution is never possible. After the students are able to explain how to add two fractions with some degree of

proficiency, ask "Why would you do it this way?" Plan this activity carefully and be prepared for some chaos.

Word Problems. Although there are those who believe that "doing word problems is problem solving," there is much more to problem solving than this. Once the algorithm is acquired for a particular type of "problem," it is no longer a problem, but rather an exercise. Problem solving is not occurring at this point. There are many good references for word problems and for methods for generating problems appropriate to the goals of the instructional unit and to assure that problem solving is taking place (see Whimbey & Lochhead, 1980, 1984; Mathematics Council of the Alberta Teachers' Association, 1982; NCTM, 1980b).

Students should be required to have complete headings, carefully defining every variable used for each problem solution. Each item in the heading should contain the words "number of." Rather than write "N = nickels," students should write "N = number of nickels," or "N = number of cents represented by the nickels." Suggest that they draw diagrams whenever possible. As the students acquire the skills to "solve" a particular type of problem (i.e., they have learned the algorithm), give them other problems using the same solution method, but varied enough to require some additional thought process. For example, make the solution a part of a larger procedure, change the "setting", or add extraneous information.

Outline the Chapter. Use this technique for introducing new material or for reviewing for a test. This is a good way for students to learn to

organize information. Some instruction in outlining methods will probably be needed.

Test Questions. Asking the student to write test questions requires not only a good understanding of the material involved but also the ability to organize and synthesize information. When the student has formulated a tentative problem, it must be evaluated to determine its suitability as a test question. This technique, utilized at the end of an instructional unit, brings together the knowledge of the unit at a high level of mental processing. Many of the higher-level cognitive processes are utilized. Problem solving and cognitive learning are taking place. Begin this technique by dividing the chapter or chapters into parts and assigning each part to one or two students. Ask each student to prepare two or three problems. Finally, evaluate the questions that have been submitted, require rewriting as necessary, and compile the questions into a review test and the official test.

Replace Two-Column Proofs with Prose. The rigor of a formal two-column proof in geometry often causes students to develop mental blocks so they are unable to complete the proof. The prose form allows students to write out their thoughts (and frustrations) and to examine and evaluate them. Because the thoughts are written down, they do not have to be held in short-term memory. This "unloading" of memory makes room for more pertinent information to be activated. After some successes with the prose form, students are usually able to organize information more effectively, and to meet with more success with the two-column proof.

Notebook. The notebook has two primary purposes: for taking daily class notes during the formal class time, and as a journal for keeping a personal record of thoughts and reflections at the end of the class period. At times the two overlap. For example, students may be asked to write out all of the steps of a problem solution in their notebooks during the class period. However, at the end of the period, they could express in their journals displeasure (or pleasure) at having to write out all of the steps of that problem solution.

During class the teacher should be posing problems at the proper level for learning to be effective. The solutions should be recorded in the notebook as examples. The lecture notes and notes on any class discussion should by rephrased in the student's own words and recorded. A method used successfully by Lubatkin (1986) has been the teacher-led class discussion to arrive at a definition of an important term. The class discussion continues until the appropriate level of precision in the definition is obtained and is understood by the class. Vocabulary lists with synonyms and definitions derived in this fashion should be recorded.

The last five to ten minutes of each class period should be devoted to journal writing. This time for writing is usually designated for reflection on what was learned in class, on a difficult concept that needs some attention, on applications of day's material, on feelings about the material, the teacher, or the classmates. An exercise, supplied by the teacher for the purpose, may be used to evaluate a critical concept. For example, after completing a unit on factoring, ask the students to discuss the following statement: "Factoring and

finding a product are reverse processes." The journal writing time gives students time to think at their own rate and to internalize new concepts by relating them to their own experiences. As students become familiar and comfortable with journal writing, their responses will show definite improvements in their organizational skills. As a result, their work becomes easier to read (Nahrgang & Petersen, 1986). Students should also be able to ask the teacher questions in the journal that may be difficult to ask in class.

Getting Started with Writing

The large number of techniques compiled in this paper may be a bit overwhelming to the teacher contemplating the introduction of writing in the mathematics classroom. What follows is an approach for getting started with a writing program.

Every student should have a notebook, and the notebook should be used every day. The teacher should develop or obtain one simple daily problem and have the students use the "explain how" or "chatter" technique on the problem during the class period. Expect to see very little real progress for two to three weeks. Recognizing and recording all the thoughts on each step in each problem takes practice. When some progress is noted, increase the complexity of the problem a little.

Notebooks are also used each day to record daily class notes. Time should be given to allow the student to rework each new definition into their own words. Examples of problems worked in class should be recorded also.

The last five to ten minutes of the class period should be reflection time for writing in the journal. Start this part as simply as possible. Ask the students to respond to the following questions:

What were the important points in today's lesson?

What did I understand from the lesson?

What did I not understand from the lesson?

What questions would I like to ask (the teacher the next day) about the lesson?

Often, when students pose these questions, they are aware of them while doing homework, and they find answers themselves. However, start the next day's class by responding to any unanswered questions.

The teacher should collect these notebooks and respond to the students' work. He or she should make appropriate comments on the written problem solutions. Students will probably demonstrate some misconceptions to the teacher in this way that would never be seen on an achievement test. This is the place to correct these misconceptions. The students may make comments or ask questions in the notebooks that they would not make or ask in class. This should become a valuable communications tool. Careful responses will be important.

As the program develops, other techniques may be added, one at a time. "Outline the Chapter" should help students arrange the topics of the chapter into a form which makes sense to them. Another good review technique is "Test Questions". The better students should benefit from writing their own questions. Time will be needed to present and develop this technique. Use the short-term techniques section above as a "shopping list" to look for ideas which may fit your students and your

style. The long-term techniques may only apply after an extended time with the writing program, and then only to the more advanced students. Start with the Writing for Learning Application as a full class review of a major unit.

The use of writing as a problem solving method brings the student into a more active role in the problem solving process and cognitive learning becomes more effective. Problem solving requires the utilization and restructuring of higher-level cognitive skills and knowledge. Actual cognitive learning may be somewhat difficult to observe and measure; however, the teacher should begin to see increased student performance on word problem solving and in other critical thinking exercises. Persistence is important as it may take some time to observe improvement in the critical thinking skills that are acquired as a result of a writing program.

Figure 6.1 General Writing Instructions

-Try to write down everything you are thinking.

-Write down each step as if you were explaining your thoughts to a friend.

-If you have done pair problem solving, think of the questioner sitting next to you, asking you to explain each step.

-Your writing is not right or wrong; it is what you are thinking at that time.

-Don't worry about spelling; try to make your writing grammatically correct so that others can understand it.

-Rephrase thoughts so that you will be sure others will understand them.

-Review the writing probes (Figure 6.2) often to look for new ideas that will help you.

-Draw diagrams whenever possible.

-Describe preliminary ideas, discuss why you accepted or rejected each one, and then explain how you expanded on the final one(s).

-When you have completed your work, go back and review it. Record your final impressions.

Figure 6.2 Writing Probes

Ideas to help you form your thoughts as you are writing.

General Probes

-What are you thinking?

-Can you say more about that?

-Explain that so that a fifth grader would understand it.

-Write that again in a different way.

-Are you sure about that?

-Check the accuracy of that last step.

-What were you thinking about to get that?

-What did you mean by "it"?

-Label that step so you can refer to it.

-What does the X stand for?

-What does that equation say in plain English?

-Could you draw a picture of that?

-Why did you write that equation?

-Tell why that last step makes sense.

When You Are Stuck

-Why are you stuck?

-What other information do you need to get unstuck?

-How can you get needed information?

-Do you need to go back and review some of your steps?

-What makes this problem difficult?

When You Think You Are Finished

-Do you have other ideas that you have not written about?

-What were you thinking about when you did step X?

-How can you show that you did this correctly?

-Could this have been done an other way (more eloquently)?

Works Cited

Anderson, J.R. (1983). The Architecture of Cognition. Cambridge, MA: Harvard Press.

Anderson, J.R. (1985). Cognitive Psychology and Its Implications. New York:W.H.Freedman and Co.

Bell, E. (1937). Men of Mathematics. New York: Simon and Schuster.

Bell, E. and Bell, R (1985). Writing and mathematical problem solving: Arguments in favor of synthesis. School Science and Mathematics, 85(3), 210-221.

Bell, F. (1978). Teaching and Learning Mathematics (In Secondary Schools). Dubuque,Iowa: Wm. C. Brown Co.

Bloom, B. and Broder, L. (1950). Problem-Solving Processes of College Students. Chicago: University Press.

Birken, M. (1986). Teaching students how to study mathematics: A classroom approach. Mathematics Teacher, 410-413.

Davis, P. and Hersh, R. (1981). The Mathematical Experience. Boston: Houghton Mifflin Co.

Fulwiler, T. (1982). Writing: An act of cognition. In C. Griffin (Ed.) Teaching Writing in All Disciplines. San Francisco: Jossey-Bass.

Gagne', E. (1985). The Cognitive Psychology of School Learning. Boston:Little, Brown and Co.

Geeslin, W. (1977). Using writing about mathematics as a teaching technique. Mathematics Teacher, 77(2), 112-115.

Johnson, L. (1985). What! Write in math class, ugh! development of a writing project for an algebra 2 class. New England Mathematics Journal, 84(11), 12-15.

Johnson, M. (1983). Writing in mathematics classes: A valuable tool for learning. Mathematics Teacher, 117-119.

Kenyon, R. (1987). Writing in the mathematics classroom. New England Mathematics Journal, 87(5), 3-19.

King, B. (1982). Using writing in the mathematics class. In C. Griffin (Ed.) Teaching Writing in All Disciplines. San Francisco: Jossey-Bass.

Konold, C. (1986). Evaluating and modifying a teaching assistant training program for a workshop-style mathematics course. Unpublished paper. Cognitive Process Research Group, University of Massachusetts.

Lochhead, J. (1985). Teaching analytic reasoning skills through pair problem solving. Segal, J.W., Chipman, S.F. and Glaser, R. (Eds.) Thinking and Learning Skills: Relating Instruction to Research. Hillsdale, NJ:Lawrence Erlbaum Associates.

Lubatkin, H. (1986). Writing is an integral part of learning math. Unpublished paper. University of Massachusetts.

Mathematics Council of the Alberta Teachers' Association (1982), Problem Solving in the Mathematics Classroom. Edmonton, Alberta: The Alberta Teachers' Association.

McMillen, L. (1986). Science and math professors are assigning writing drills to focus students' thinking. The Chronicle of Higher Education, Jan. 22, 1986.

Mett, C. (1987). Writing as a learning device in calculus. Mathematics Teacher, 80, 534-537.

Nahrgang, C. and Petersen, B. (1986). Using writing to learn mathematics. Mathematics Teacher, 461-465.

National Council of Teachers of Mathematics (1980a). An Agenda for Action: Recommendations for School Mathematics of the 1980s. Reston, VA: Author.

National Council of Teachers of Mathematics (1980b). Problem Solving in School Mathematics. Reston, VA: Author.

Newman, J. (1956). The World of Mathematics. New York: Simon and Schuster.

Papert, S. (1980). Mindstorms: Children, Computers, and Powerful Ideas. New York: Basic Books, Inc.

Polya, G. (1957). How To Solve It. Princeton: Princeton University Press.

Salem, J. (1982). Using writting in teaching mathematics. In Bart, M; D'Arcy, P. and Healy, M. What's Going On?. Montclair, New Jersey: Boynton/Cook.

Schoenfeld, A. (1982). Recent advances in mathematics education: Ideas and implications. In S. Rachlin (Ed.) Problem Solving in the Mathematics Classroom. Edmonton, Alberta: The Alberta Teachers' Association.

Schoenfeld, A. (1985). Mathematical Problem Solving. New York: Academic Press, Inc.

Tchudi, S. and Yates, J. (1983). Teaching Writing in the Content Areas: Senior High School. Washington: National Education Association.

Whimbey, A. and Lochhead, J. (1980). Problem Solving and Comprehension: A Short Course in Analytical Reasoning (2nd ed.). Philadelphia: Franklin Institute Press.

Whimbey, A. and Lochhead, J. (1984). Beyond Problem Solving and Comprehension: An Exploration of Quantitative Reasoning. Philadelphia: Franklin Institute Press.

Wickelgren, W. (1974) How to Solve Problems - Elements of a Theory of Problems and Problem Solving. San Francisco: W.H. Freeman and Co.

Wolfe, D. and Reising, R. (1983). Writing for Learning in the Content Area. Portland Maine: J. Weston Walch.